CIVL.666, MANE.666
FUNDAMENTALS OF FINITE ELEMENTS
HOMEWORK 10
Due: November 15, 2019

Please turn that day – Want to return the homework in the next class since test 2 could be as early as a week after that.

1. (to be graded) The goal of this problem is to construct the stiffness matrix and body load vector for a 1-D element using cubic shape functions. The first two shape functions are linear interpolating shape functions in terms of the end values. The second two shape functions are hierarchic quadratic and cubic shape functions as given in class. Explicitly write out the four shape functions and evaluate the stiffness matrix and load vector for a weak form given as:

$$\int_{-1}^{1} (w_{,\xi} u_{,\xi} + wu)\, d\xi = \int_{-1}^{1} w(2-\xi)\, d\xi$$

The material below provides expressions needed to construct the shape functions. Also note that the coordinates of the domain of the weak form and the parametric coordinates of the element are the same so there is no Jacobian term to worry about.

$$N_1 = \frac{1-\xi}{2} \quad ; \quad N_2 = \frac{1+\xi}{2} \quad ; \quad N_{j+1} = \phi_j(\xi),\ j = 2(1)p$$

$$\phi_j(\xi) = \frac{1}{\sqrt{2(2j-1)}}\left(P_j(\xi) - P_{j-2}(\xi)\right)$$

$$P_0(\xi) = 1 \ ;\ P_1(\xi) = \xi \ ;\ P_2(\xi) = \frac{1}{2}(3\xi^2 - 1)$$

$$P_3(\xi) = \frac{1}{2}(5\xi^3 - 3\xi) \ ;\ P_4(\xi) = \frac{1}{8}(35\xi^4 - 30\xi^2 + 3)$$

$$P_n(\xi) = \frac{(2n-1)}{n}\xi P_{n-1}(\xi) - \frac{(n-1)}{n}P_{n-2}(\xi)$$

2. State the interelement continuity requirements and, using the standard convergence rate formula (pg. 190 of the text), indicate the rates of convergence in the $H^0, H^1 \cdots H^m$ norms for the following four pairs of classes of problem (*m*) and polynomial order elements (*k*).

   $m = 1, k = 4 \qquad m = 3, k = 2 \qquad m = 2, k = 2 \qquad m = 3, k = 6$

3. For each of the elements in question 2 that are useful (would converge), indicate, if possible, a set of nodal degrees of freedom for a 1-D element that could be used with the selected k and meet no higher than the minimum interelement continuity requirements dictated by the given m. (It is possible that in some cases more that one set of nodal degrees of freedom that will meet the required conditions, while in others none may be possible.)