CIVL.666, MANE.666 FUND. OF FINITE ELEMENTS
HOMEWORK 3
Due: September 24, 2019

1. (to be graded) Consider the strong problem statement: Find u such that

$$-\kappa u_{,xx} - \lambda u + 2x^2 = 0 \qquad 0 < x < 1$$

subject to

$$u(0) = 1$$
$$u(1) = -2$$

Derive the weak form of this problem appropriate for a Galerkin discretization (κ and λ are in general functions of x for this part). Solve this problem employing the given quadratic shape functions. In this part assume κ and λ are constants with values of $\kappa=2$ and $\lambda=3$. The three shape functions that play a role are the same as before, that is

$$N_1 = 1 - 3x + 2x^2, N_2 = 4x(1-x), N_3 = x(2x-1)$$

(Note - be sure to use the correct shape function(s) with the degrees of freedom and with the essential boundary condition(s)).

2. Consider the strong problem statement: Given κ is a constant, find u such that

$$\kappa u_{,xxxx} + f = 0 \qquad 0 < x < 1$$

subject to

$$u(0) = g_1$$
$$u_{,x}(0) = g'$$
$$u(1) = g_2$$
$$\kappa u_{,xx}(1) = h$$

Derive the weak form of this problem appropriate for a Galerkin discretization.

In constructing your weak form be sure to explicitly state the conditions on the trial and weighting functions and be sure to show those boundary terms that are zero and those for which the natural boundary conditions are represented by substitution in the correct place(s) in the weak form.

Like the beam this will require two integrations by parts and the terms picked-up along that help deal with natural boundary conditions. Also as in the beam example there is a pair of boundary conditions at each end. Those proportional to the value of the function and its first derivative are essential boundary conditions and those proportional to the second and third derivatives are natural boundary conditions. As we typically do, we will select our trial functions to satisfy the essential boundary conditions a-priori, but not state anything about the natural boundary conditions being met a-priori and make sure they are represented in the final weak form.