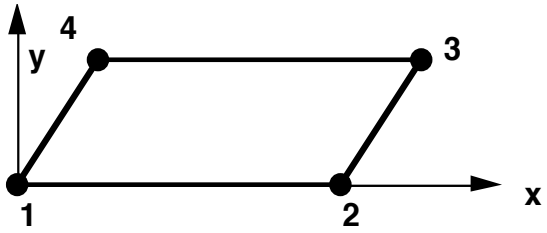
CIVL.666, MANE.666 FUNDAMENTALS OF FINITE ELEMENTS
HOMEWORK 6
Due: Oct. 18, 2019

1. Discuss the suitability of the shape functions given below for satisfying the intra-element continuity (C1) condition and the basic completeness condition (C3) as required for the use of the element for 2-D heat transfer analysis.

$$u^h = a_1 x + a_2 y + a_3\left(xy - \frac{x_4}{y_4}y^2\right) + a_4$$

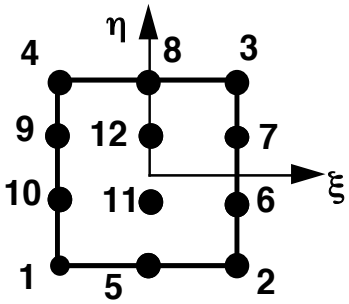
where x_4 and y_4 are the coordinates of node 4.
Note: $x_3=x_2+x_4$ and $y_3=y_4$ (the element is a parallelogram)

Hint: It is straight forward to answer the two parts of this question working directly with the shape functions in the x-y system as given above. (Trying to put into a nodal interpolant form will not work – they do not correspond to nodal interpolant shape functions). The inter-element continuity requirement (which you do not need to do for homework) is not so straight forward in this case unless one made the assumption that nodes shared between elements are shared by both elements and move together in which case one can look at the variation of the displacements between them.



2. Exercise 1. on Page 128.

3. (to be graded) Derive a set of shape functions associated with nodes 2, 10, 12 for the 12 node (quadratic-cubic Lagrangian) element that satisfies the C^0 inter-element continuity requirements (condition C2). The unknowns in the formulation are to be the values of the function at the node points. Note that the element is a bi-unit square and nodes are equally spaced in the two coordinate directions.



4. Demonstrate satisfaction of the inter-element continuity requirement (C2) for the shape functions for the 9-noded quadrilateral discussed in example 4 on pages 129-130.