

CIVL.666, MANE.666
FUNDAMENTALS OF FINITE ELEMENTS
HOMEWORK 9
Due: Nov. 12, 2019

- Exercise 7 on page 161 of the text. Part a. only. Some hints: Remember to substitute the change of coordinates to integrate the area in the parametric coordinates. A one-point integration rule will exactly integrate terms with up to linear contributions in each direction. More specifically, a term $\xi\eta$ will be integrated exactly since it is linear in ξ and linear in η . However, a term ξ^2 would not be integrated exactly by a one-point rule since it is quadratic in one of the directions.
- (to be graded) Construct the element stiffness matrix for the 1-D heat transfer element given below the following three ways: (i) exactly integrating, (ii) numerical integrating using a trapezoidal rule (see pg. 140 of text), and (iii) numerically integrating using two point Gaussian quadrature (see pg. 141-142 of text).

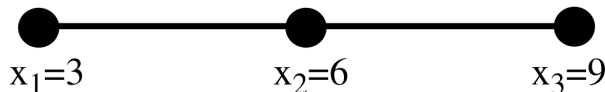
$$N_1 = \frac{1}{2}\xi(\xi - 1), \quad N_2 = 1 - \xi^2, \quad N_3 = \frac{1}{2}\xi(\xi + 1)$$

where the shape functions are defined on the bi-unit element in the parametric coordinates.

For the 1-D heat transfer problem (assuming a constant conductivity of 1.0) the stiffness terms are defined as:

$$K_{ab}^e = \int_{\Omega^e} N_{a,x} N_{b,x} dx$$

The domain of the element for this example is defined by the position of the three nodes as given in the figure below. Assume an isoparametric element and properly account for the coordinate transformation process. Compare and comment on the resulting stiffness matrices.



- Would the process of calculating the exact element stiffness terms for the element in problem 2 become more difficult if the middle node was placed at a location other than the middle of the element (e.g., $x_2=7$)? Please explain your answer. (Note – you are not asked to actually do the integration.)
- Develop a three-point Newton-Cotes (approximate the integrand by Lagrange polynomials) for a 1-D element with a parametric coordinate system goes from 0 to 1. The locations of the integration points are to be at 0.25, 0.5 and 0.75.