Your NAME ____

Fundamentals of Finite Elements, Test 2, November 26, 2019

Test is a closed book test.

Problem 1 (20 pts)	________	Problem 2 (15 pts)	_________
Problem 3 (25 pts)	________	Problem 4 (20 pts)	_________
Problem 5 (20 pts)	________	Total (100pts)	________

$$\frac{\partial}{\partial\xi_i} = \frac{\partial x}{\partial\xi_i}\frac{\partial}{\partial x} + \frac{\partial y}{\partial\xi_i}\frac{\partial}{\partial y} \ , (uv)_{,x} = u_{,x}v + uv_{,x} \ , \frac{d}{d\xi} = \frac{dx}{d\xi}\frac{d}{dx} = J(\xi)\frac{d}{dx}$$

$$\int f(x)\delta(x - x_b)dx = f(x_b) \ , \int_a^b uv_{,x}dx = uv\Big|_a^b - \int_a^b u_{,x}v dx \ , \int_{\Omega^e} f(x)dx = \int_{-1}^{1} f(x(\xi))\frac{dx}{d\xi}d\xi$$

$$N_a(\xi) = \prod_{i=1}^{3} N_i^a(\xi_i) \ , \quad N_i^a(\xi_i) = \begin{cases} \displaystyle\prod_{J=1}^{I_i^a} \frac{m\xi_i - J + 1}{J} & \text{for } I_i^a \geq 1 \\ 1 \ \text{for } I_i^a = 0 \end{cases} \ , I_i^a = m\xi_i\Big|_{\tilde{\xi} = \tilde{\xi}_a}$$

$$\|e\|_s = ch^{\beta}\|u\|_{k+1} \ , \beta = min(k + 1 - s, 2(k + 1 - m))$$

$$l_a^{n_{en}-1} = \frac{\displaystyle\prod_{\substack{b=1 \\ b\neq a}}^{n_{en}} (\xi - \xi_b)}{\displaystyle\prod_{\substack{b=1 \\ b\neq a}}^{n_{en}} (\xi_a - \xi_b)} \ , \sin\left(\frac{\pi}{2}\right) = \cos(0) = 1 \ , \cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{-\pi}{2}\right) = 0$$

$$\cos(\pi) = -1 \ , \sin(0) = \sin(\pi) = 0 , (w, f) = \int_{\Omega} wf d\Omega \ , (w, h)_{\Gamma} = \int_{\Gamma_h} wh d\Gamma$$

$$e = u^h - u \ , a(w^h, e) = 0 \text{ for all } w^h \in V^h \ , a(e, e) \leq a(U^h - u, U^h - u) \text{ for all } U^h \in \delta^h$$

For the case of $\delta^h = V^h$ we have $a(u, u) = a(u^h, u^h) + a(e, e)$

For the case of $\delta^h = V^h$ we have $a(u^h, u^h) \leq a(u, u)$

$$|J| = \frac{\partial x}{\partial\xi}\frac{\partial y}{\partial\eta} - \frac{\partial x}{\partial\eta}\frac{\partial y}{\partial\xi} \quad , \quad \frac{d}{d\xi} = \frac{dx}{d\xi}\frac{d}{dx} \quad , \quad \int_a^b f(\xi)d\xi \approx \sum_{i=1}^{n} f(\xi_i)\int_a^b l_i^{n-1}\, d\xi = \sum_{i=1}^{n} f(\xi_i)W_i$$